Rutgers Geometric Analysis Conference – November 14-16, 2018

To register as a participant (required), apply for funding (still available), or view further details of the program, please visit the conference website:

https://finmath.rutgers.edu/geometric-analysis-conf-2018

This event is supported by Rutgers University and the National Science Foundation. Graduate students, junior faculty, women, individuals from groups underrepresented in the mathematical sciences or from institutions with little federal support, and persons with disabilities are especially encouraged to participate and to apply for support.

Organizers

Natasa Sesum and *Paul Feehan*
Rutgers, The State University of New Jersey,
New Brunswick

Conference Schedule

Wednesday

For Wednesday mini course titles and abstracts (talks intended for junior mathematicians), please see alphabetical last name list below under "Mini course Talk Titles and Abstracts"

Morning session in Hill 705

9:00-10:00 Theodora Bourni: Minicourse - Introduction to mean curvature flow I
10:10-11:10 Theodora Bourni: Minicourse - Introduction to mean curvature flow II
11:10-11:20 Coffee break
11:20-12:20 Robert Haslhofer: Minicourse - Ricci curvature and stochastic analysis I

12:20-1:30 Lunch break at Busch Campus Faculty/Staff Dining Hall (participants on their own)

Afternoon session in Hill 705

1:30-2:30 Robert Haslhofer: Minicourse - Ricci curvature and stochastic analysis II
2:40-3:40 Yevgeny Liukomovich: Minicourse - Minimal surfaces and quantitative topology of the space of cycles I
3:40-3:50 Coffee break
3:50-4:50 Yevgeny Liukomovich: Minicourse - Minimal surfaces and quantitative topology of the space of cycles II

For Thursday and Friday research titles and abstracts, please see alphabetical last name list below under "Research Talk Titles and Abstracts"

Thursday

Morning session in Hill 705

9:00-9:50 Theodora Bourni
10:00-10:50 Yevgeny Liokumovich
10:50-11:00 Coffee break in Hill 703
11:00-11:50 Valentino Tosatti

12:00-1:30 Lunch break at Busch Campus Faculty/Staff Dining Hall (participants on their own)

Afternoon session in Hill 705 and Core 401

2:00-2:50 Chris Gerig (in Hill 705)
3:00-3:50 Tristan Rivière (in Hill 705)
3:50-4:20 Coffee break outside Core 401
4:20-5:10 Thomas Walpuski (in Core 401)
5:20-6:10 Yu Wang (in Core 401)

Friday

Morning session in Core 101

9:00-9:50 Alice Chang
10:00-10:50 Matthew Gursky
10:50-11:00 Coffee break outside Core 101
11:00-11:50 Robert Haslhofer

12:00-1:30 Lunch break at Busch Campus Faculty/Staff Dining Hall (participants on their own)

Afternoon session in Hill 116

1:40-2:30 Camillo De Lellis
2:40-3:30 Alessandro Carlotto
3:30-3:50 Coffee break
3:50-4:40 Nicholas Edelen

Thursday and Friday Research Talk Titles and Abstracts

Theodora Bourni

Title: Ancient Pancakes

Abstract: We construct a compact, convex ancient solution of mean curvature flow in $\mathbb{R}^{n+1}$ with $O(1)\times O(n)$ symmetry that lies in a slab of width π. We provide detailed asymptotics for this solution and show that, up to rigid motions, it is the only compact, convex, $O(n)$-invariant ancient solution that lies in a slab of width π and in no smaller slab. This work is joint with Mat Langford and Giuseppe Tinaglia.

Alessandro Carlotto

Title: New results on the geometric convergence of minimal subvarieties

Abstract: A fundamental theorem, due to Choi and Schoen, asserts that the space of (closed) minimal surfaces of fixed topology inside a compact 3-manifold of positive Ricci curvature is compact in the sense of smooth graphical convergence with multiplicity one. This sort of conclusion is patently false when working in higher-dimension, in fact even in the round four-dimensional sphere (cf. Hsiang). Much more subtle is, instead, the problem of recovering sharp geometric compactness results in ambient dimension three under weaker curvature conditions, and in particular under the sole assumption that the ambient *scalar* curvature be positive. I will describe the state of the art of this subject, how it connects with the classification of low-index complete minimal surfaces in the Euclidean space, and to some key open problems.

Alice Chang

Title: A conformally invariant gap theorem characterizing $\mathbb{C}\mathbb{P}^2$ via the Ricci flow

Abstract: I will report some recent joint work with Matt Gursky and Siyi Zhang. We extend the sphere theorem to give a conformally invariant characterization of ($\mathbb{C}\mathbb{P}^2, g_{FS}$). In particular, we introduce a conformal invariant $\beta(M^4, [g]) \geq 0$ defined on conformal four-manifolds satisfying a 'positivity' condition; it follows that if $0 \leq \beta(M^4, [g]) < 4,$ then M^4 is diffeomorphic to S^4. Our main result of this paper is a 'gap' result showing that if $b^+_2 (M^4) > 0$ and $4 \leq \beta(M^4, [g]) < 4 + \epsilon$ for $\epsilon > 0$ small enough, then M^4 is diffeomorphic to $\mathbb{C}\mathbb{P}^2$. The Ricci flow is used in a crucial way to pass from the bounds on β to pointwise curvature information. I will also report some related gap theorems on Bach flat metrics by Siyi Zhang.

Camillo De Lellis

Title: Rigidity and flexibility of isometric embeddings

Abstract: Consider a smooth connected closed two-dimensional Riemannian manifold Σ with positive Gauss curvature. If u is a C^2 isometric embedding of Σ, then $u (\Sigma)$ is convex. On the other hand, in the fifties Nash and Kuiper showed, astonishingly, that this conclusions is in general false for C^1 isometric embeddings. It is expected that the threshold at which isometric embeddings "change nature" is the $\frac{1}{2}$-Hoelder continuity of their derivatives, a conjecture which shares a striking similarity with another famous one in the theory of fully developed turbulence. In my talk I will review several plausible reasons for the threshold and a very recent work, joint with Dominik Inauen, which indeed shows a suitably weakened form of the conjecture.

Nicholas Edelen

Title: The structure of minimal surfaces near polyhedral cones

Abstract: We prove a $C^{1,\alpha}$-regularity theorem for stationary varifolds which resemble a cone $\bC_0^2$ over an equiangular geodesic net. For varifold classes admitting a ``no-hole'' condition on the singular set, we additionally establish $C^{1,\alpha}$-regularity near the cone $\bC_0^2 \times \R^m$. Combined with work of Allard, Simon, Taylor, and Naber-Valtorta, our result implies a $C^{1,\alpha}$-structure for the top three strata of minimizing clusters and size-minimizing currents, and a Lipschitz structure on the $(n-3)$-stratum.

Chris Gerig

Title: SW = Gr

Abstract: Whenever the Seiberg-Witten (SW) invariants of a 4-manifold X are defined, there exist certain 2-forms on X which are symplectic away from some circles. When there are no circles, i.e. X is symplectic, Taubes' "SW=Gr"" theorem asserts that the SW invariants are equal to well-defined counts of J-holomorphic curves (Taubes' Gromov invariants). In this talk I will describe an extension of Taubes' theorem to non-symplectic X: there are well-defined counts of J-holomorphic curves in the complement of these circles, which recover the SW invariants. This "Gromov invariant"" interpretation was originally conjectured by Taubes in 1995. This talk will involve contact forms.

Matthew Gursky

Title: Obstructions to the existence of conformally compact Einstein manifolds

Abstract: I will talk about a singular boundary value problem for Einstein metrics that arises in the Fefferman-Graham theory of conformal invariants and in the AdS/CFT correspondence. I will describe an index-theoretic invariant which gives an obstruction to existence in the case of spin manifolds. This is joint work with Q. Han and S. Stolz. Since existence is obstructed, I will talk about recent work with R. Graham and G. Szekelyhidi on some related singular boundary value problems that are unobstructed.

Robert Haslhofer

Title: Ancient low entropy flows, mean convex neighborhoods, and uniqueness

Abstract: In this talk, I will describe our proof of the mean convex neighborhood conjecture for the mean curvature flow of surfaces in $\mathbb{R}^3$. Namely, if the flow has a spherical or cylindrical singularity at a space-time point $X=(x,t)$, then there exists a positive $\varepsilon=\varepsilon(X)>0$ such that the flow is mean convex in a space-time neighborhood of size ε around X. The major difficulty is to promote the infinitesimal information about the singularity to a conclusion of macroscopic size. In fact, we prove a more general classification result for all ancient low entropy flows that arise as potential limit flows near X. Namely, we prove that any ancient, unit-regular, cyclic, integral Brakke flow in $\mathbb{R}^3$ with entropy at most $\sqrt{2\pi/e}+\delta$ is either a flat plane, a round shrinking sphere, a round shrinking cylinder, a translating bowl soliton, or an ancient oval. As an application, we prove the uniqueness conjecture for mean curvature flow through spherical or cylindrical singularities. In particular, assuming Ilmanen's multiplicity one conjecture, we conclude that for embedded two-spheres the mean curvature flow through singularities is well-posed. This is joint work with Kyeongsu Choi and Or Hershkovits.

Yevgeny Liokumovich

Title: Weyl law for the volume spectrum

Abstract: To each cohomology class of the space of codimension 1 cycles, Min-Max theory associates a minimal hypersurface with some integer multiplicity. Volumes of these minimal hypersurfaces are called "widths" or "volume spectrum" of the manifold. Gromov conjectured that like eigenvalues of the Laplacian, the volume spectrum has asymptotic behaviour described by a Weyl law. I will discuss the proof of this conjecture, related open questions and directions. This is a joint work with Fernando Coda Marques and Andre Neves.

Tristan Rivière

Title: A proof of the multiplicity one conjecture for minmax minimal surfaces in arbitrary codimensions

Abstract: Given any admissible k-dimensional family of immersions of a given closed oriented surface into an arbitrary closed Riemannian manifold, we prove that the corresponding min-max width for the area is achieved by a smooth (possibly branched) immersed minimal surface with multiplicity one and Morse index bounded by k.

Valentino Tosatti

Title: Estimates for collapsing Ricci-flat metrics

Abstract: I will discuss a priori estimates for families of Ricci-flat metrics on a compact Calabi-Yau manifold which fibers over a lower-dimensional base, as the size of the fibers shrinks to zero. These metrics are obtained by solving a family of complex Monge-Ampere equations with ellipticity degenerating in the fiber directions. I will present C^infty a priori estimates when the fibers are isomorphic to each other, and C^alpha estimates in general. The new technical tools are sharp new Schauder estimates for the Laplacian on cylinders, and nonlinear Liouville theorems on cylinders. This is joint work with H.-J. Hein.

Thomas Walpuski

Title: Existence of harmonic $\mathbb{Z}/2$\mathbb{Z}/$ spinors via wall-crossing

Abstract: Recently, Taubes introduced the notion of harmonic harmonic $\mathbb{Z}/2$\mathbb{Z}/$ spinors and initiated the study of their singular sets. These are special cases of harmonic multi-valued spinors; arguably the simplest instance being harmonic multi-valued 1-forms. They appear naturally in the compactifications of number of gauge-theoretic PDE, for example: the Kapustin-Witten equation and the Seiberg–Witten equation with two spinors. Although, various compactness theorems "predict" the existence of harmonic Z/2Z spinors they do not actually guarantee it. The purpose of this talk is to explain a concrete existence result. Our proof is of a topological nature; at its heart lies a wall-crossing formulae for the Seiberg-Witten equation with two spinors. This is joint work with Aleksander Doan.

Yu Wang

Title: Sharp estimate of global Coulomb gauge in dimension 4

Abstract: Consider a principle SU(2)-bundle P over a compact 4-manifold M and a $W^{1,2}$-connection A of P satisfying $\|F_A\|_{L^2(M)}$. Our main result is the existence of a global section $\sigma: M \to P$ with controllably many singularities such that the connection form σ^*A satisfies the Coulomb equation $d^*(\sigma^*A) = 0$ and moreover admits a sharp estimate. In this talk, we first recall some preliminaries and then outlines the proof. If time allows we shall elaborate on the ideas to overcome the main difficulties in this problem, which include an ϵ-regularity theorem for the Coulomb gauge, an annular-bubble regions decomposition for the curvature, and studying the singularity behavior of the Coulomb gauge on each annular and bubble region.

Wednesday Mini-course Talk Titles and Abstracts

Theodora Bourni

Title 1: Minicourse - Introduction to mean curvature flow I

Abstract 1: Mean curvature flow is the gradient flow of the area functional; it moves the surface in the direction of steepest decrease of area. An important motivation for the study of mean curvature flow comes from its potential geometric applications. One reason for this is that it preserves several natural curvature inequalities, an observation that has led to its use as a means of proving deep classification theorems.

In the first lecture we will introduce the one-dimensional version of the mean curvature flow, namely the curve shortening flow. We will discuss certain basic properties of this flow, such as short time existence and the evolution equations of important geometric quantities. Moreover we will give an overview of an incredible result of Grayson that states that any simple regular closed curve contracts to a point in finite time.

Titles 2: Minicourse - Introduction to mean curvature flow Ii

Abstract 2: We will introduce mean curvature flow of hypersurfaces in Euclidean space. As in lecture one, we will briefly mention short and long-time existence as well as the evolution equations of certain geometric quantities. We will discuss Huisken's famous monotonicity formula and show how it can be used to deduce interesting properties on the shape of singularities.

Robert Haslhofer

Title 1: Minicourse - Ricci curvature and stochastic analysis I

Abstract 1: I'll describe a new link between geometry and probability. First, I'll give a brief introduction to Ricci curvature, the Einstein equations and Hamilton's Ricci flow. Next, I'll give a quick introduction to Brownian motion in Euclidean space and on manifolds, and integration by parts on path space. Finally, I'll explain joint work with Aaron Naber where we discovered an infinite-dimensional Bochner formula for martingales on path space, which vastly generalizes the classical Bochner formula for the heat flow on manifolds. Using these ideas, we can make sense of solutions of the Einstein equations and the Ricci flow in the setting of singular spaces.

Title 2: Minicourse - Ricci curvature and stochastic analysis II

Abstract 2: I'll describe a new link between geometry and probability. First, I'll give a brief introduction to Ricci curvature, the Einstein equations and Hamilton's Ricci flow. Next, I'll give a quick introduction to Brownian motion in Euclidean space and on manifolds, and integration by parts on path space. Finally, I'll explain joint work with Aaron Naber where we discovered an infinite-dimensional Bochner formula for martingales on path space, which vastly generalizes the classical Bochner formula for the heat flow on manifolds. Using these ideas, we can make sense of solutions of the Einstein equations and the Ricci flow in the setting of singular spaces.

Yevgeny Liokumovich

Title 1: Minicourse - Minimal surfaces and quantitative topology of the space of cycles I

Abstract 1: The purpose of this minicourse is to give a simple and accessible introduction to the geometry and topology of the space of flat cycles in Riemannian manifolds and applications to existence results for minimal surfaces. We will start by reviewing papers of Almgren, Gromov and Guth, and then discuss in detail the proof of the Weyl law for the volume spectrum and its applications to density and equidistribution results for minimal hypersurfaces.

Title 2: Minicourse - Minimal surfaces and quantitative topology of the space of cycles II

Abstract 2: The purpose of this minicourse is to give a simple and accessible introduction to the geometry and topology of the space of flat cycles in Riemannian manifolds and applications to existence results for minimal surfaces. We will start by reviewing papers of Almgren, Gromov and Guth, and then discuss in detail the proof of the Weyl law for the volume spectrum and its applications to density and equidistribution results for minimal hypersurfaces.